

Analytical studies of Spectrum Broadcast Structures in Quantum Brownian Motion

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Abstract. Spectrum Broadcast Structures are a new and fresh concept in the quantum-to-classical transition, introduced recently in the context of decoherence and the appearance of objective features in quantum mechanics. These are specific quantum state structures, responsible for an apparent objectivity of a decohered state of a system. Recently they have been shown to appear in the well known Quantum Brownian Motion model, however the final analysis relied on numerics. Here, after a presentation of the main concepts, we perform analytical studies of the model, showing the timescales and the efficiency of the spectrum broadcast structure formation. We consider a massive central system and a somewhat simplified environment being random with a uniform distribution of the frequencies.

1. Introduction

A measurement in quantum mechanics typically alters a state of the system, so that if several observers try to measure a certain observable they will in general interfere with each other. This is in stark contrast with classical world, where properties of systems such as position, momentum, etc can be in principle observed by as many observers as one wishes, they will all agree on the results (modulo eventual reference frame transformations), and moreover will not disturb the system. This observer-independence and non-disturbance may be taken as a basis of an intuitive definition of objectivity. Thus a problem arises: How can one explain the observed objectivity of everyday world with quantum mechanics? It can be seen as one of the aspects of the quantum-to-classical transition—a problem present from the very beginning of quantum mechanics [1].

One of the prominent attempts to address this problem has been known as quantum Darwinism [2]. It is a refined and more realistic version of decoherence theory (see e.g. [3, 4, 5]), where one realizes that often observations are made indirectly, through portions of the environment rather than by a direct interaction with the object (e.g. an illuminated object scatters photons, which are then detected by the eyes of the observers). Hence, the environment is no longer treated merely as a source of noise and dissipation but is recognized as an important "information carrier". This implies a paradigmatic shift in the main object of studies in the theory of open quantum systems—from the reduced state of the system ϱ_S alone [3, 4, 5] to a joint state of the system and an observed fraction fE of the environment $\varrho_{S:fE}$. Objectivity is then linked to "information redundancy": If the environment acquires in the course of the decoherence a large number of copies of the state of the system and this information can be read out without disturbance, then the state of the system becomes objective [2]. In pictorial terms, objective becomes information about the system which not only "survives" the interaction with the environment, but manages to proliferate in the latter. As a measure of this effect a family of scalar criteria based on the quantum mutual information $I(\varrho_{S:fE})$ between the system and a fraction of the environment as a function of this fraction size (e.g. a number of scattered photons taken into account in fE). If at some point no change is produced while increasing the fraction size (and the so called classical plateau appears on partial information plots), it is then concluded that environment stores a largely redundant amount of information about the system state (more precisely about pointer states to which the system decoheres).

This approach has been reconsidered in [6] on a ground that scalar criteria, and quantum mutual information in particular, might not be the right tools and more convincing arguments for information proliferation are needed, preferably on the most fundamental level available—that of quantum states. Such an approach has indeed been proposed in [6] (for a somewhat complementary approach see [7]). Starting from an intuitive definition of objectively existing state of the system [2, 8] it has been shown

that, under certain assumptions, it singles out a structure of a partially reduced state of the system and the observed fraction of the environment compatible with objectivity to the following one, called Spectrum Broadcast Structure (SBS):

$$\varrho_{S:fE} = \sum_i p_i |i\rangle \langle i| \otimes \varrho_i^{E_1} \otimes \dots \otimes \varrho_i^{E_M}, \quad \varrho_i^{E_k} \varrho_{i' \neq i}^{E_k} = 0, \quad (1)$$

where $|i\rangle$ is the pointer basis to which the system decoheres, p_i are the initial pointer probabilities and $\varrho_i^{E_k}$ are some states of the fragments of the environment $E_1, \dots, E_M \in fE$, which are supposed to be observed and hence cannot be traced out. The information about the state of the system— the index i , is encoded in a number of copies $1, \dots, M$ in the environment through the states $\varrho_i^{E_k}$ and can be perfectly recovered due to the assumed non-overlap condition in (1) and without any disturbance (on average) to the whole state $\varrho_{S:fE}$. By the foregoing discussion this leads to an apparent objectivity of the state of the system. Quite surprisingly the converse is also true as shown in [6] with a help of several assumptions. One of them is that "non-disturbance" requirement should be understood in the sense of Bohr's reply [9] to the famous EPR paper [10] and which has been further formalized in [11]. The other important assumption is a, so called, strong independence condition, demanding that the only correlation between parts of the environment is due to the common information about the system. Formally, from quantum information point of view, states (1) realize a certain weak form of state broadcasting [12], called spectrum broadcasting [13]: The spectrum p_i of the reduced state of the system ϱ_S is present in many copies in the environments E_1, \dots, E_M and can be retrieved from there by projections on the supports of $\varrho_i^{E_k}$ (due to the non-overlap condition in (1)). This broadcasting process can be also described by a channel [13]:

$$\Lambda^{S \rightarrow fE}(\varrho_{0S}) = \sum_i \langle i | \varrho_{0S} | i \rangle \varrho_i^{E_1} \otimes \dots \otimes \varrho_i^{E_M}, \quad (2)$$

where ϱ_{0S} is the initial state of the system. Thus, objectivity can be seen as a result of a certain broadcasting process given by the channel (2).

The power of the above result is that it links objectivity and quantum state structures in a completely abstract, model independent way. A natural question then arises if those structures appear in the canonical models of decoherence [3]: collisional decoherence, Quantum Brownian Motion (boson-boson), spin-spin and spin-boson models. So far the first two models were analyzed and the answer is in general affirmative: There exists parameter regimes of the models such that SBS are formed. The first studied example was the famous illuminated sphere model of collisional decoherence of Joos and Zeh [14]. Following the quantum Darwinism inspired analysis of [15], it has been shown in [16] that indeed spectrum broadcast structures (1) are asymptotically formed in the course of the evolution, even if the environment is noisy (initially in a mixed state) and the appropriate time scales were given. Important, from the perspective of the current work, are the methods introduced for checking for the SBS, which will be reviewed in the next chapter. The sphere model, however illustrative, is rather simple since the system has no self-dynamics. More realistic, and richer, in this sense is Quantum Brownian Motion, where a central harmonic oscillator

is linearly coupled to a bath of harmonic oscillators. This is arguably one of the most popular models describing quantum dissipative systems. Despite its long history [17], only recently studies of the informational content of the environment has appeared [18, 19, 20, 21]. The first two works analyzed both numerically and analytically (in the massive central system regime) the scalar condition of quantum Darwinism, assuming initially pure environment and squeezed state of the system and showing that indeed the characteristic classical plateau is being formed. On the other hand in [20, 21] the model has been analyzed from the SBS perspective and, under somewhat similar conditions as above but with thermal environment, a numerical evidence have been found that indeed there are parameter regimes so that a SBS is formed. A distinctive feature of the found structure is that it is dynamical and evolves in time: The pointer basis in (1) rotates in time according to the self-Hamiltonian of the central oscillator and at any instant a SBS is being formed, encoding traces of this motion. One has to stress that due to the mentioned paradigmatic shift in the treatment of the environment, i.e. it may contain useful information, one cannot assume it to be so inert as not to feel the presence of the system, as it is done in the usual Born-Markov approximation and master equation approach to open quantum systems (see e.g. [3] for an introduction and standard applications). Thus, in particular our study of Quantum Brownian Motion does not rely on the Born-Markov approximation and master equation methods, but rather on a direct state analysis (the details are presented in Section 3). The drawback of the previous studies [20] is that the analysis of the SBS formation was at the end performed numerically. Here, continuing the previous research, we overcome this difficulty and show analytically that there is a parameter regime of the model such that a dynamical SBS is formed. We give analytical expressions for the decoherence and the SBS formation time-scales in both low- and high-temperature regimes.

2. Checking for Spectrum Broadcast Structures

The method of detection of SBS developed so far [16, 21] is rather direct and most naturally apply to the situation when the system-environment interaction is of the von Neumann measurement type:

$$\hat{H}_{SE} = \hat{X} \otimes \sum_{k=1}^N \hat{Y}_k, \quad (3)$$

where \hat{X} , \hat{Y}_k are some observables of the system and the k -th environment respectively, assumed for simplicity to have discrete spectra. Albeit of a special form, this class of Hamiltonians is of a fundamental importance both for the decoherence [3] and measurement [23] theories and thus worth investigating. To illustrate the method, we will neglect here the self-Hamiltonians of the system and the environment (quantum measurement limit) as one can then calculate everything explicitly. The resulting unitary evolution given by (3) is of a controlled unitary type, where the system controls the

environments through eigenvalues ξ of \hat{X} :

$$\hat{U}(t) = \sum_{\xi} |\xi\rangle \langle \xi| \otimes \hat{U}_1(\xi; t) \otimes \cdots \otimes \hat{U}_N(\xi; t), \quad \hat{U}_k(\xi; t) \equiv e^{-i\xi t \hat{Y}_k / \hbar}. \quad (4)$$

Assuming, as it is usually done, a fully product initial state $\varrho_{0S} \otimes \varrho_{01} \otimes \cdots \otimes \varrho_{0k}$, one immediately obtains that after the tracing of some portion of the environment, denoted $(1-f)E$ and containing a fraction fN , $0 < f < 1$ subsystems, the state reads:

$$\varrho_{S:fE}(t) = \text{tr}_{(1-f)E} \left[\hat{U}(t) \varrho_{0S} \otimes \bigotimes_{k=1}^N \varrho_{0k} \hat{U}(t)^\dagger \right] \quad (5)$$

$$= \sum_{\xi} \langle \xi | \varrho_{0S} | \xi \rangle | \xi \rangle \langle \xi | \otimes \bigotimes_{k=1}^{fN} \hat{U}_k(\xi; t) \varrho_{0k} \hat{U}_k(\xi; t) \quad (6)$$

$$+ \sum_{\xi \neq \xi'} \Gamma_{\xi, \xi'}(t) \langle \xi | \varrho_{0S} | \xi' \rangle | \xi \rangle \langle \xi' | \otimes \bigotimes_{k=1}^{fN} \hat{U}_k(\xi; t) \varrho_{0k} \hat{U}_k(\xi'; t), \quad (7)$$

where:

$$\Gamma_{\xi, \xi'}(t) \equiv \prod_{k \in (1-f)E} \text{tr} \left[\hat{U}_k(\xi; t) \varrho_{0k} \hat{U}_k(\xi'; t) \right] = \prod_{k \in (1-f)E} \text{tr} \left[\varrho_{0k} e^{-i(\xi - \xi') t \hat{Y}_k / \hbar} \right] \quad (8)$$

is the usual decoherence factor between the states $|\xi\rangle$, $|\xi'\rangle$. A check for the SBS (1) proceeds in two steps.

First of all the coherent part (7), containing entanglement between the system and the environment, should vanish and this is of course the usual decoherence process, controlled by $\Gamma_{\xi, \xi'}(t)$. If one is able to show $|\Gamma_{\xi, \xi'}(t)| = 0$ with time for all different pair of ξ, ξ' , one proves that decoherence has taken place and $|\xi\rangle$ becomes the pointer basis.

Second, we check if the information deposited in the environment during the decoherence can be perfectly read out, i.e. if the system-dependent states of the environments:

$$\varrho_{\xi k}(t) \equiv U_k(\xi; t) \varrho_{0k} U_k(\xi; t) \quad (9)$$

have non-overlapping supports (cf. (1)):

$$\varrho_{\xi k}(t) \varrho_{\xi' k}(t) = 0, \quad (10)$$

and hence are perfectly one-shot distinguishable. Among different measures of distinguishability [22], the most suitable turns out to be the generalized overlap

$$B(\varrho_1, \varrho_2) \equiv \text{tr} \sqrt{\sqrt{\varrho_1} \varrho_2 \sqrt{\varrho_1}}. \quad (11)$$

This is due to the fact that most interesting are the situations when the interaction with each individual portion of the environment in (3) is vanishingly small (see e.g. [14]). Then, one cannot expect (10) to hold at the level of single environments. Right to the contrary, since each of the unitaries $\hat{U}_k(\xi; t)$ weakly depends on the parameter ξ , the states $\varrho_{\xi k}(t)$ are almost identical for different ξ 's. In other words, the information about ξ is diluted in the environment. However, it can happen that by grouping subsystems of the observed part of environment fE into larger fractions, called macrofractions *mac*

and introduced in [16], one can approach the perfect distinguishability (10) at the level of macrofraction states $\varrho_\xi^{mac}(t) \equiv \bigotimes_{k \in mac} \varrho_{\xi k}(t)$. Generalized overlap is well suited for such tests due to its factorization with the tensor product:

$$B_{\xi, \xi'}^{mac}(t) \equiv B(\varrho_\xi^{mac}(t), \varrho_{\xi'}^{mac}(t)) = \prod_{k \in mac} B(\varrho_{\xi k}(t), \varrho_{\xi' k}(t)). \quad (12)$$

Summarizing, if one is able to prove that for some time both functions vanish

$$|\Gamma_{\xi, \xi'}(t)| \approx 0, \quad B_{\xi, \xi'}^{mac}(t) \approx 0, \quad (13)$$

then from (6,7) this is equivalent to the formation of the spectrum broadcast structure (1):

$$\varrho_{S:FE} \approx \sum_{\xi} p_{\xi} |\xi\rangle \langle \xi| \otimes \varrho_{\xi}^{mac_1} \otimes \dots \otimes \varrho_{\xi}^{mac_M}, \quad (14)$$

with $\varrho_{\xi}^{mac_k}$ having orthogonal supports for different ξ 's and the convergence is in the trace norm.

The introduced above grouping into macrofractions (or equivalently coarse-graining of the observed environment) can be seen as a reflection of detection thresholds of real-life detectors, e.g. an eye. Since one is usually interested in a thermodynamic-type of a limit $N \rightarrow \infty$ it is important that those fractions scale with N (hence the name "macrofractions").

3. Spectrum Broadcast Structures in Quantum Brownian Motion

The model Hamiltonian [4, 5, 17] reads:

$$\hat{H} = \frac{\hat{P}^2}{2M} + \frac{M\Omega^2 \hat{X}^2}{2} + \sum_{k=1}^N \left(\frac{\hat{p}_k^2}{2m_k} + \frac{m_k \omega_k^2 \hat{x}_k^2}{2} \right) + \hat{X} \sum_{k=1}^N C_k \hat{x}_k, \quad (15)$$

where \hat{X}, \hat{P} are the position and momentum of the central oscillator of mass M and frequency Ω , \hat{x}_k, \hat{p}_k are the positions and momenta of the bath oscillators, each with mass m_k and frequency ω_k , and C_k are the coupling constants. This model can be in principle solved explicitly either directly [17] or using Wigner functions [24]. However, as unlike in the standard treatments we are interested here not merely in the reduced state of the central oscillator alone, but in the joint state of the central and a part of bath oscillators, the mentioned exact methods do not produce manageable solutions. As already stated in the Introduction, the standard master equation methods are of no use either, since we are primarily interested in the influence of the system on the environment and not the other way around. Following this "inverted" logic, one can try to eliminate, at least in the first approximation, the recoil on the system due to the environment (apart from the renormalization of the frequency). This suggest a greatly simplifying assumption of a massive central system [18, 19], which we will adopt. One can then use a non-adiabatic version of the Born-Oppenheimer (Non-Born-Oppenheimer, NBO) approximation (see e.g. [25]), where the central system evolves unperturbed, according to its self-Hamiltonian $\hat{H}_S = \hat{P}^2/(2M) + M\Omega^2 \hat{X}^2/2$ (with the renormalized frequency

$\Omega^2 \equiv \Omega_{bare}^2 - \sum_k C_k^2 / (2m_k \omega_k^2)$ and the environment follows this evolution in the following way. The system propagator $K_t(X; X_0) \equiv \langle X | e^{-i\hat{H}_S t/\hbar} | X_0 \rangle$ is rewritten with the help of the classical trajectory $X(t; X_0)$, starting at $t = 0$ at X_0 and reaching X at time t (as it is well known for the oscillator this semi-classical approximation is exact; see e.g. [26]) and this trajectory acts as a classical driving force for the environment through the coupling term, leading to a controlled evolution:

$$i\hbar \frac{\partial}{\partial t} |\psi_E(t)\rangle = \hat{H}_E(X(t; X_0)) |\psi_E(t)\rangle, \quad (16)$$

where $\hat{H}_E(X(t; X_0)) \equiv \sum_{k=1}^N [\hat{p}_k^2 / (2m_k) + m_k \omega_k^2 \hat{x}_k^2 / 2] + X(t; X_0) \sum_{k=1}^N C_k \hat{x}_k$. The full system-environment state is then constructed using the Born-Oppenheimer type of an ansatz:

$$\Psi_{S:E}^{NBO}(X, \mathbf{x}) = \int dX_0 \phi_{S0}(X_0) \langle X | e^{-i\hat{H}_S t/\hbar} | X_0 \rangle \langle \mathbf{x} | \hat{U}_E(X(t; X_0)) | \psi_{E0} \rangle, \quad (17)$$

with $|\phi_{S0}\rangle, |\psi_{E0}\rangle$ being the initial states of the system and the environment respectively, and $\hat{U}_E(X(t; X_0))$ is a solution of (16).

From the type of the coupling in (15) and the analysis of the previous Section it follows that the candidates for the pointer states will be related to the position eigenstates. Hence, initial states of the system with large coherences in the position are of the greatest interest and for the purpose of this study we choose an initially momentum-squeezed ground state as the initial state of the system (studies of the initially position-squeezed state from [20] suggest that there is no SBS formation; cf. [19]). This reduces the classical trajectories to $X(t; X_0) = X_0 \cos \Omega t$ [18, 19] and in this case the evolution (17) can be formally re-written using [20]:

$$\hat{U}_{S:E}(t) = \int dX_0 e^{-i\hat{H}_S t/\hbar} | X_0 \rangle \langle X_0 | \otimes \hat{U}_E(X_0 \cos \Omega t). \quad (18)$$

The driven evolution of the environment is easily solved and gives [20]:

$$\hat{U}_E(X_0 \cos \Omega t) = \bigotimes_{k=1}^N e^{i\zeta_k(t) X_0^2} e^{-i \sum_k \hat{H}_k t/\hbar} \hat{D}(\alpha_k(t) X_0) \equiv \bigotimes_{k=1}^N \hat{U}_k(X_0; t), \quad (19)$$

where $\zeta_k(t)$ is a phase factor (unimportant for our considerations), $\hat{H}_k \equiv \hat{p}_k^2 / (2m_k) + m_k \omega_k^2 \hat{x}_k^2 / 2$, $\hat{D}(\alpha) \equiv e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$ is the displacement operator, and:

$$\alpha_k(t) \equiv -\frac{C_k}{2\sqrt{2\hbar m_k \omega_k}} \left[\frac{e^{i(\omega_k + \Omega)t} - 1}{\omega_k + \Omega} + \frac{e^{i(\omega_k - \Omega)t} - 1}{\omega_k - \Omega} \right]. \quad (20)$$

The evolution (19) is formally a controlled-unitary type (4), in which the environment evolves accordingly to the initial position X_0 of the central oscillator. To study our central object—the partially reduced state (5), one should be careful with the integral in (18) as not to lose the diagonal part (6). We rewrite the integral using a finite division of the real line of X_0 into intervals $\{\Delta_i\}$ with $|X_0\rangle \langle X_0|$ replaced by orthogonal projectors $\hat{\Pi}_\Delta$ on the intervals Δ (continuous distribution of X_0 is recovered in the limit of these divisions see e.g. [27]). The partially traced state then reads:

$$\varrho_{S:fE}(t) = \sum_{\Delta} e^{-i\hat{H}_S t/\hbar} \hat{\Pi}_\Delta |\phi_0\rangle \langle \phi_0| \hat{\Pi}_\Delta e^{i\hat{H}_S t/\hbar} \bigotimes_{k=1}^{fN} \varrho_k(X_\Delta; t) \quad (21)$$

$$+ \sum_{\Delta \neq \Delta'} \Gamma_{X_\Delta, X_{\Delta'}}(t) e^{-i\hat{H}st/\hbar} \hat{\Pi}_\Delta |\phi_0\rangle \langle \phi_0| \hat{\Pi}_{\Delta'} e^{i\hat{H}st/\hbar} \otimes \bigotimes_{k=1}^{fN} \hat{U}_k(X_\Delta; t) \varrho_{0k} \hat{U}_k(X_{\Delta'}; t)^\dagger,$$

where X_Δ is some position within an interval Δ ,

$$\varrho_k(X_0; t) \equiv \hat{U}_k(X_0; t) \varrho_{0k} \hat{U}_k(X_0; t)^\dagger \quad (22)$$

are the system-dependent states of the environments (9), and:

$$\Gamma_{X_0, X'_0}(t) \equiv \prod_{k \in (1-f)E} \text{tr} \left[\hat{U}_k(X_0; t) \varrho_{0k} \hat{U}_k(X'_0; t)^\dagger \right] \equiv \prod_{k \in (1-f)E} \Gamma_{X_0, X'_0}^{(k)}(t), \quad (23)$$

is the decoherence factor. Following the general procedure of Section 2, one has to calculate it together with the generalized overlap (11) for the states (22): $B_{X_0, X'_0}^{(k)}(t) \equiv B(\varrho_k(X_0; t), \varrho_k(X'_0; t))$ as they serve as the indicator functions for the formation of spectrum broadcast structures; cf. (13). Assuming the environment oscillators are initially in the thermal states with the same temperature, the form of the decoherence factor has been known [5]:

$$-\log |\Gamma_{X_0, X'_0}^{(k)}(t)| = \frac{(X_0 - X'_0)^2}{2} |\alpha_k(t)|^2 \coth(\tau_T \omega_k) = \quad (24)$$

$$\begin{aligned} & \frac{|X_0 - X'_0|^2 C_k^2 \omega_k \coth(\tau_T \omega_k)}{4\hbar m_k (\omega_k^2 - \Omega^2)^2} \left[(\cos \omega_k t - \cos \Omega t)^2 - \left(\sin \omega_k t - \frac{\Omega}{\omega_k} \sin \Omega t \right)^2 \right] \\ & \equiv \frac{|X_0 - X'_0|^2}{2} f_T^\Gamma(t; \omega_k) \end{aligned} \quad (25)$$

where $\tau_T \equiv \hbar/(2k_B T)$ is the thermal time and we have introduced a function $f_T^\Gamma(t; \omega_k) \equiv |\alpha_k(t)|^2 \coth(\tau_T \omega_k)$ for a later convenience. The generalized overlap, in turn, for thermal (and also more general Gaussian) environments has been obtained in [20]:

$$\begin{aligned} -\log B_{X_0, X'_0}^{(k)}(t) &= \frac{(X_0 - X'_0)^2}{2} |\alpha_k(t)|^2 \tanh(\tau_T \omega_k) \\ &\equiv \frac{|X_0 - X'_0|^2}{2} f_T^B(t; \omega_k). \end{aligned} \quad (26)$$

We note that the factor $\coth(\tau_T \omega_k)$ appearing in the decoherence factor is related to the mean initial energy of the environmental oscillators at temperature T , $\coth(\tau_T \omega_k) = \langle E(\omega, T) \rangle / E_0(\omega)$, where $E_0(\omega) \equiv \hbar\omega/2$ is the zero-point energy, while $\tanh(\tau_T \omega_k)$, appearing in the generalized overlap, is nothing else but the purity $\text{tr}(\varrho_{0k}^2)$ of the initial thermal state ϱ_{0k} , which in turn is related to the linear entropy $S_{lin}(\varrho_{0k}) = 1 - \text{tr}(\varrho_{0k}^2)$. Thus, the effectiveness of the decoherence depends on the initial energy of the environment, while information accumulation on its purity.

To proceed further with the analysis, one has to specify the environment. The standard procedure [3, 4, 5, 17, 18, 19] is to pass to a continuum limit of frequencies ω_k and encode the properties of the environment in a specific continuous approximation to the spectral density function $J(\omega) = \sum_k C_k^2 / (2m_k \omega_k) \delta(\omega - \omega_k)$ (e.g. in [18, 19] the Ohmic spectral density has been chosen). In contrast, in [20] a somewhat different idea has been put forward: To keep the environment discrete but random, with frequencies ω_k chosen

from some given ensemble. Randomness is needed to effectively induce decoherence in the spirit of [28], as the environment remains finite-dimensional. For definiteness' sake the simplest case has been studied, where the frequencies ω_k are independently, identically distributed (i.i.d.) with a uniform distribution over a finite interval $[\omega_L, \omega_U]$. The interval is chosen so that the environment is off-resonant (cf. (24,26)), to avoid decohereing of the system by a single environment, and "fast":

$$\omega_U, \omega_L \gg \Omega. \quad (27)$$

This choice of the environment may be considered as a direct, "mechanistic", as opposed to the usual field, treatment of the environment: The bath is a collection of identical mechanical oscillators with masses m_k and random frequencies ω_k . It leads to a complication in the study of the conditions (13) as from (23, 24) and (12, 26) the macroscopic indicator functions, associated with the traced over part of the environment $(1-f)E$ and an observed macrofraction mac respectively:

$$|\Gamma_{X_0, X'_0}(t)| = \exp \left[-\frac{|X_0 - X'_0|^2}{2} \sum_{k \in (1-f)E} f_T^\Gamma(t; \omega_k) \right], \quad (28)$$

$$B_{X_0, X'_0}(t) = \exp \left[-\frac{|X_0 - X'_0|^2}{2} \sum_{k \in mac} f_T^B(t; \omega_k) \right] \quad (29)$$

become almost periodic functions of time. Previously [20] those functions were studied only numerically, indicating that indeed there is a parameter regime that the time averages over very long times of the above (non-negative) functions simultaneously vanish, indicating small typical fluctuations above zero. This in turn, implies that the partially traced state (21) approaches dynamical spectrum broadcast form with respect to the initial position X_0 :

$$\begin{aligned} \varrho_{S:fE}(t) &\approx \int dX_0 |\langle X_0 | \phi_0 \rangle|^2 e^{-i\hat{H}st} |X_0\rangle \langle X_0| e^{i\hat{H}st} \otimes \\ &\otimes \varrho_{mac_1}(X_0; t) \otimes \cdots \otimes \varrho_{mac_{\mathcal{M}}}(X_0; t), \end{aligned} \quad (30)$$

with $\varrho_{mac_k}(X_0; t)$ having non-overlapping supports. The dynamical character of the above structure is manifest in the time dependence of all the appearing states: The pointer basis is rotating according to the system self-Hamiltonian (thanks to the recoil-free assumption) $|X(t)\rangle \equiv e^{-i\hat{H}st} |X_0\rangle$ and this motion modulates the evolution of the environment in such a way that an instantenous SBS is formed at any moment. In a sense, the environment encodes the motion of the central oscillator. The main parameters this process depends on through (28,29) are time, temperature, separation $\Delta X_0 \equiv |X_0 - X'_0|$ [21], and macrofraction size N_{mac} . Trade-offs between them dictate if and when the structure (30) will be formed. In what follows we study this behavior analytically, assuming large macrofraction size N_{max} .

4. Analytical estimates of the SBS formation

As mentioned in the previous Section, we are working with random environments with i.i.d. frequencies ω_k with some distribution $P(\omega)$. As a consequence, the functions $f_T^B(t; \omega_k)$ and $f_T^\Gamma(t; \omega_k)$, appearing in the SBS indicator functions (28,29), also become i.i.d. random variables for a fixed time t and temperature T . Analytical study of their sums over a macrofraction $\sum_{k=1}^{N_{mac}} f_T^{\Gamma,B}(t; \omega_k)$ (we assume for simplicity that both the unobserved macrofraction $(1-f)E$ as well as each of the observed ones have the same size N_{mac}) is possible in the limit of a large macrofraction size $N_{mac} \rightarrow \infty$ using the Law of Large Numbers (LLN) [29]. This will be our main tool. It states (in its strong form) that the macrofraction averages $1/N_{mac} \sum_{k=1}^{N_{mac}} f_T^{\Gamma,B}(t; \omega_k)$ converge almost surely, i.e. with probability one, to their expectation values:

$$\frac{1}{N_{mac}} \sum_{k=1}^{N_{mac}} f_T(t; \omega_k) \xrightarrow{a.s.} \int d\omega P(\omega) f_T(t; \omega) \equiv \langle\langle f_T(t; \omega) \rangle\rangle \quad (31)$$

(we will be neglecting the superscripts Γ, B unless it leads to a confusion) and, according to the large deviation theory, the probability of error is exponentially small in N_{mac} with the rate governed by the, so called, rate function (which we will not be interested in here, only assuming that it exists and is non-zero). This allows us to approximate the sums $\sum_{k=1}^{N_{mac}} f_T(t; \omega_k)$ with $N_{mac} \langle\langle f_T(t; \omega) \rangle\rangle$.

We note that the invocation of LLN is, in this context, effectively equivalent to the continuous limit for the macrofractions of the environment with $P(\omega)$ determining the spectral density. In other words, we divide the environment into fractions of such a large size that the LLN may be applied. Following our approach, explained in the previous Section, instead of the standard spectral densities, such as e.g. Ohmic, we will use here a much simpler, uniform probability distribution over an interval $[\omega_L, \omega_U]$ due to an ease of analysis:

$$\langle\langle f_T(t; \omega) \rangle\rangle = \frac{1}{\Delta\omega} \int_{\omega_L}^{\omega_U} d\omega f_T(t; \omega), \quad (32)$$

where $\Delta\omega = \omega_U - \omega_L$. In what follows we analyze the short- and long-time behavior of this expression in the limits of high and low temperature. This will enable us to estimate the macrofraction size N_{mac} needed in order for the functions (28,29) to attain asymptotically values close to zero within a given error as well as give the timescales of their initial decays, observed numerically in [20].

4.1. Low temperature

Let us first assume that the temperature is so low, that the associated thermal energy is much lower than the lowest oscillator energy: $k_B T \ll \hbar\omega_L$. Then in the leading order the temperature dependence can be neglected $\coth(\hbar\omega_k/2k_B T) \approx \tanh(\hbar\omega_k/2k_B T) \approx 1$ and the behavior of decoherence and orthogonalization becomes identical:

$$f_T^\Gamma(t; \omega_k) \approx f_T^B(t; \omega_k) \approx |\alpha_k(t)|^2 \equiv f_0(t; \omega_k) \quad (33)$$

with $\alpha_k(t)$ given by (20). The calculation of the ensemble mean of $f_0(t; \omega_k)$ is rather lengthy and is presented in Appendix C, with an assumption that the interaction strengths C_k obey (cf. [18, 19]) $C_k = 2\sqrt{(Mm_k\bar{\gamma}_0)/\pi}$, with $\bar{\gamma}_0$ a constant.

First, we are interested in the short-time behavior, valid for times much shorter than the shortest timescale of the full Hamiltonian, which in this case is $t \ll \omega_U^{-1}$ (we recall that we assume Ω to be much lower than the environmental frequencies in order to be in the off-resonant regime, so that collections rather than individual environments matter). By expanding the expression for $\langle\langle f_0(t; \omega) \rangle\rangle$ in power series with respect to time, we find after a tedious calculation that (for details see Appendix C eq. (C.10)):

$$\langle\langle f_0(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} \log\left(\frac{\omega_U}{\omega_L}\right) t^2 + O(t^4), \quad (34)$$

which immediately implies that the initial behavior of both the decoherence and the orthogonalization factors is a Gaussian decay (c.f. (24, 26)):

$$|\Gamma_{X_0, X'_0}(t)| \approx B_{X_0, X'_0}(t) \approx \exp\left[-N_{mac} \left(\frac{t}{\tau_0}\right)^2\right], \quad (35)$$

with a common timescale:

$$\frac{\tau_0}{\sqrt{N_{mac}}}, \quad \tau_0 = \frac{\hbar\pi\Delta\omega}{\Delta X_0 M \bar{\gamma}_0} \log^{-1}\left(1 + \frac{\Delta\omega}{\omega_L}\right). \quad (36)$$

We note that it depends on the macrofraction size and the separation through the product $\Delta X_0 \sqrt{N_{mac}}$. Thus, in order to keep the same time-scale for small separations the macrofraction size should increase quadratically with decreasing separation.

The initial Gaussian decay (35) by no means guarantees that the functions will stay close to zero with negligible fluctuations—revivals are possible, as has been shown in [20]. Thus a long-time analysis is needed, governed in our case by the condition $t \gg 1/(\omega_L - \Omega) \approx 1/\omega_L$ as $\Omega \ll \omega_L$. The detailed calculation is tedious and is given in the Appendix C, eq. (C.11). The result reads:

$$\langle\langle f_0(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} (A_0 \cos^2(\Omega t) + B_0), \quad (37)$$

where:

$$A_0 \equiv -\frac{1}{2\Omega^2} \left(2 \log \frac{\omega_U}{\omega_L} - \log \frac{\omega_U^2 - \Omega^2}{\omega_L^2 - \Omega^2}\right), \quad (38)$$

$$B_0 \equiv \frac{1}{\omega_L^2 - \Omega^2} - \frac{1}{\omega_U^2 - \Omega^2} - A_0. \quad (39)$$

$$(40)$$

Interestingly, for large times the mean has an oscillatory part with the system frequency Ω , but for fast environments (27) this part is vanishingly small as $A_0 \approx 0$. The above formulas allow us to solve a very important problem in the context of SBS: How big should be choose macrofractions in order to get decoherence and orthogonalization with a prescribed error ϵ (common in the low T limit for both functions):

$$|\Gamma_{X_0, X'_0}(t)|, B_{X_0, X'_0}(t) < \epsilon. \quad (41)$$

This in turn will determine the trace norm distance of the actual state $\varrho_{S;fE}(t)$ to the spectrum broadcast form. From (37) and (28,29) we immediately obtain that if:

$$\Delta X_0^2 N_{mac} > \frac{\hbar\pi\Delta\omega}{M\bar{\gamma}_0 B_0} \log \frac{1}{\epsilon} \approx \frac{\hbar\pi\omega_U^2\omega_L^2}{M\bar{\gamma}_0(\omega_U + \omega_L)} \log \frac{1}{\epsilon}, \quad (42)$$

then the functions will be bounded by (41) for all times $t \gg 1/(\omega_L - \Omega)$. This result can be treated as an analytical proof of SBS formation in the studied regime. Similarly to the short-time decay (35), the asymptotic behavior of $|\Gamma_{X_0, X'_0}(t)|$, $B_{X_0, X'_0}(t)$ is governed by the product $\Delta X_0^2 N_{mac}$, so that the increase of the macrofraction size is quadratic with decreasing the spatial resolution of the SBS. This finite spatial resolution of the SBS for a given error level and a macrofraction size is a manifestation of the "macroscopic objectivity" idea, introduced in [21] for simplified models of QBM. Namely, for a given tolerance ϵ and a macrofraction size, the objective state of the system appear only on the length scales greater than ones given by (42).

4.2. High temperature

Here we consider the opposite situation of a hot environment: $k_B T \gg \hbar\omega_U$. Intuitively, a formation of the SBS should be quite compromised now, as high temperature, while increasing the decoherence power of the environment through the increase of its energy appearing in (24), decreases its information capacity, by decreasing the purity, on which depends the orthogonalization factor (26). Indeed, this is what we show below. In the leading order $\tanh(\tau_T\omega) = [\coth(\tau_T\omega)]^{-1} \approx \tau_T\omega$ and (24) and (26) read:

$$f_T^\Gamma(t; \omega_k) \approx \frac{1}{\tau_T\omega_k} |\alpha_k(t)|^2, \quad (43)$$

$$f_T^B(t; \omega_k) \approx \tau_T\omega_k |\alpha_k(t)|^2. \quad (44)$$

The relevant means (32) can be calculated analytically again; see Appendix D.1 and Appendix D.2. For short time-scales $t \ll \omega_U^{-1}$ we obtain the following behavior (for the details see Appendix D, eq. (D.12), (D.23)):

$$\langle\langle f_T^\Gamma(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\omega_L\omega_U\tau_T} t^2 + O(t^4), \quad (45)$$

$$\langle\langle f_T^B(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi} \tau_T t^2 + O(t^4), \quad (46)$$

resulting again in the initial Gaussian decay:

$$|\Gamma_{X, X'}(t)| \approx \exp \left[-N_{mac} \left(\frac{t}{\tau_{dec}} \right)^2 \right] \quad (47)$$

$$B_{X, X'}(t) \approx \exp \left[-N_{mac} \left(\frac{t}{\tau_{ort}} \right)^2 \right]. \quad (48)$$

However, this time the timescales are different. For the decoherence one obtains (cf. (D.12))

$$\frac{\tau_{dec}}{\sqrt{N_{mac}}}, \quad \tau_{dec} = \tau_T \frac{\hbar\pi\omega_L\omega_U}{\Delta X_0 M\bar{\gamma}_0}, \quad (49)$$

whereas for generalized overlap (cf. (D.23)) the characteristic time is:

$$\frac{\tau_{ort}}{\sqrt{N_{mac}}}, \quad \tau_{ort} = \tau_T^{-1} \frac{\hbar\pi}{\Delta X_0 M \bar{\gamma}_0}. \quad (50)$$

As one would expect, the key difference is in the temperature dependence through the thermal time $\tau_T = \hbar/(2k_B T)$. While τ_{dec} decreases as T^{-1} indicating faster decoherence with higher temperature, $\tau_{ort} \sim T$ so that it may even happen that the orthogonalization timescale $\tau_{dec}/\sqrt{N_{mac}}$ is larger than the validity of the short-time approximation $t \ll \omega_U^{-1}$. Keeping $\tau_{dec}/\sqrt{N_{mac}} < \omega_U^{-1}$ so that the short-time approximation, and hence the Gaussian decay, is valid, puts a constraint on the temperature, the macrofraction size and the separation to be discriminated:

$$\frac{T}{\Delta X_0 \sqrt{N_{mac}}} < \frac{M \bar{\gamma}_0}{2\pi k_B \omega_U}. \quad (51)$$

To get some insight into possible revivals of the decoherence and orthogonalization factors, we perform long-time analysis. In Appendix D it is shown that for $t \gg 1/(\omega_L - \Omega) \approx 1/\omega_L$ the asymptotic expression for $\langle\langle f_T^\Gamma(t; \omega) \rangle\rangle$ reads:

$$\langle\langle f_T^\Gamma(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega\tau_T} (A_\Gamma \cos^2(\Omega t) + B_\Gamma) + O(t^{-1}) \quad (52)$$

with:

$$A_\Gamma \equiv -\frac{1}{4\Omega^2} \left[\frac{\Delta\omega}{\omega_U \omega_L} + \frac{1}{2\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right],$$

$$B_\Gamma \equiv \frac{1}{4\Omega^2} \left(\frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} \right) - A_\Gamma, ,$$

while for generalized overlap it is:

$$\langle\langle f_T^B(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi\Delta\omega} (A_B \cos^2(\Omega t) + B_B) + O(t^{-1}), \quad (53)$$

where:

$$A_B \equiv \frac{1}{2\Omega} \log \frac{(\omega_U - \Omega)(\omega_L + \Omega)}{(\omega_L - \Omega)(\omega_U + \Omega)} \quad (54)$$

$$B_B \equiv \frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2}. \quad (55)$$

We observe that unlike in the low T regime, the decoherence asymptotic keeps oscillating with the system frequency Ω even for fast environments (27) as $A_\Gamma \approx \Delta\omega/(4\Omega^2\omega_U\omega_L)$, while $A_B \approx 0$. We are now ready to solve the problem of the SBS formation in the high temperature regime: In a given temperature T , how big should be the macrofraction sizes to achieve decoherence and distinguishability, and hence the SBS, on a length-scale ΔX_0 within given errors:

$$|\Gamma_{X_0, X'_0}(t)| < \epsilon_{dec}, \quad B_{X_0, X'_0}(t) < \epsilon_{ort} \quad (56)$$

Eqs. (52) and (53) give us the answer:

$$T\Delta X_0^2 N_{mac}^\Gamma > \frac{\hbar^2\pi\Delta\omega}{2Mk_B\bar{\gamma}_0 B_\Gamma} \log \frac{1}{\epsilon_{dec}} \approx \frac{\hbar^2\pi\Omega^2\omega_U\omega_L}{Mk_B\bar{\gamma}_0} \log \frac{1}{\epsilon_{dec}}, \quad (57)$$

$$\frac{\Delta X_0^2 N_{mac}^B}{T} > \frac{2\pi k_B \Delta\omega}{M\bar{\gamma}_0 B_B} \log \frac{1}{\epsilon_{ort}} \approx \frac{2\pi k_B \omega_U \omega_L}{M\bar{\gamma}_0} \log \frac{1}{\epsilon_{ort}}, \quad (58)$$

where N_{mac}^I is the size of the traced-over part of the environment $(1 - f)E$ and N_{mac}^B is the size of (each of) the observed macrofraction. As predicted, keeping all other parameters fixed, the observed macrofraction size in high temperature must be much larger than the unobserved one in order to come close to SBS. Indeed, from the above results those sizes scale like the thermal-to-central-system energies:

$$\frac{N_{mac}^B}{N_{mac}^I} > 2 \left(\frac{k_B T}{\hbar \Omega} \right)^2 \frac{\log \epsilon_{ort}}{\log \epsilon_{dec}} \quad (59)$$

and the later factor is huge for the considered fast environments, since $k_B T \gg \hbar \omega_U \gg \hbar \Omega$.

5. Conclusions

We have studied the process of formation of the spectrum broadcast structures in Quantum Brownian Motion model, continuing the research initiated in [20]. Being interested in the information gained by the environment about the system, we have considered a rather non-standard limit of a massive central system (initially in the momentum squeezed state), and somewhat simplified random environments with i.i.d. uniformly distributed frequencies. The use of the Law of Large Numbers, assuming the environment to be sufficiently large, allowed us to obtain analytical results on the spectrum broadcast structure formation.

In particular, we have investigated short-time behavior of decoherence and generalized overlap factors in low and high temperatures. In the low temperature regime, we have shown that both factors admit Gaussian decay with the same timescale, which depends on frequencies of unobserved environmental oscillators only. In the high temperature regime they also decay in a Gaussian way. However the resulting timescales are different functions of temperature: the decoherence rate is proportional to the temperature, whereas the rate of decay of generalized overlap is inversely proportional to temperature. This explains in a quantitative way previous numerical simulations showing rapid decoherence and vanishing orthogonalization of remaining environmental states in the studied model with growing temperature for fixed number of environmental systems.

Long-time analysis gave us the efficiency of the spectrum broadcast structure formation in the sense of the required observed/unobserved macrofraction sizes to obtain decoherence and the environmental state distinguishability within given errors. In low temperatures these sizes are equal, but, as one would expect, in high temperature they have the opposite temperature dependence as hot environments decohere the central system efficiently but encode a vanishingly small amount of information due to high noise.

An obvious generalization of the present work would be an analysis of more standard environment models, e.g. the Ohmic one with a cut-off. The resulting functions will be more complicated but we believe still analyzable, at least in certain approximate regimes, similar to the studied above.

6. Acknowledgements

We would like to thank J. Wehr, P. Horodecki, and R. Horodecki for discussions and remarks. JKK acknowledges the financial support of the John Templeton Foundation through the grant ID #56033. JT acknowledges support of the Polish National Science Center by means of project no. 2015/16/T/ST2/00354 for the PhD thesis.

Appendix A. Appendix Content

Appendix B is devoted to Sine and Cosine Integrals. In Appendix B.1 we introduce notation for particular combinations of Sine and Cosine integrals that appear in formulas. In Appendix B.2 and Appendix B.3 formulas for short-time and long-time behavior of this functions are presented. Appendix C contains details of computations for low-temperature Quantum Brownian Motion. The high-temperature case is treated in Appendix D with Appendix D.1, Appendix D.2 devoted to decoherence factor and generalized overlap respectively.

Appendix B. Sine and Cosine integrals

Appendix B.1. Notation

It will prove beneficial for the sake of clarity to introduce the following notation for combinations of Sine and Cosine integrals:

$$F_{\text{Si}}(\pm, \pm, \pm, \pm) = [\pm 1, \pm 1, \pm 1, \pm 1] \cdot [\text{Si}((\omega_L - \Omega)t), \text{Si}((\omega_U - \Omega)t), \text{Si}((\omega_L + \Omega)t), \text{Si}((\omega_U + \Omega)t)]^T \quad (\text{B.1})$$

$$F_{\text{Ci}}(\pm, \pm, \pm, \pm) = [\pm 1, \pm 1, \pm 1, \pm 1] \cdot [\text{Ci}((\omega_L - \Omega)t), \text{Ci}((\omega_U - \Omega)t), \text{Ci}((\omega_L + \Omega)t), \text{Ci}((\omega_U + \Omega)t)]^T, \quad (\text{B.2})$$

where $[\pm 1, \dots, \pm 1]$ is a vector, \cdot denotes vector product and T stands for transposition. The argument of $F_{\text{Si}}(\pm, \pm, \pm, \pm)$, $F_{\text{Ci}}(\pm, \pm, \pm, \pm)$ specifies signs' pattern of functions e.g.:

$$F_{\text{Si}}(+, -, +, -) = \text{Si}((\omega_L - \Omega)t) - \text{Si}((\omega_U - \Omega)t) + \text{Si}((\omega_L + \Omega)t) - \text{Si}((\omega_U + \Omega)t)$$

Appendix B.2. Short-time behavior

In the short time regime, i.e. for $t \ll \omega_U^{-1}$, we can approximate relevant functions as follows:

$$\begin{aligned} F_{\text{Si}}(+, -, +, -) &= 2(\omega_L - \omega_U)t + \frac{t^3}{9}(\omega_U^3 - \omega_L^3 + 3\Omega^2\omega_U - 3\Omega^2\omega_L) + O(t^5) \\ F_{\text{Si}}(+, -, -, +) &= \frac{t^3}{3}\Omega(\omega_L^2 - \omega_U^2) + O(t^5) \\ F_{\text{Ci}}(+, -, +, -) &= \log \frac{\omega_L^2 - \omega^2}{\omega_U^2 - \omega^2} + \frac{1}{2}(\omega_U^2 - \omega_L^2)t^2 + o(t^4) \\ F_{\text{Ci}}(+, -, -, +) &= \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} + \Omega(\omega_L - \omega_U)t^2 + O(t^4), \quad (\text{B.3}) \end{aligned}$$

Appendix B.3. Long-time behavior

On the other hand, the asymptotic of relevant functions is given by:

$$tF_{\text{Si}}(+, -, +, -) = 2\left(\omega_U \frac{\cos(\omega_U t)}{\omega_U^2 - \Omega^2} \cos(\Omega t) + \Omega \frac{\sin(\omega_U t)}{\omega_U^2 - \Omega^2} \sin(\Omega t) - \right.$$

$$\begin{aligned}
& \omega_L \frac{\cos(\omega_L t)}{\omega_L^2 - \Omega^2} \cos(\Omega t) - \Omega \frac{\sin(\omega_L t)}{\omega_L^2 - \Omega^2} \sin(\Omega t) \Big) + O(t^{-1}) \\
tF_{\text{Si}}(+, -, -, +) &= 2 \Big(\omega_U \frac{\sin(\omega_U t)}{\omega_U^2 - \Omega^2} \sin(\Omega t) + \Omega \frac{\cos(\omega_U t)}{\omega_U^2 - \Omega^2} \cos(\Omega t) - \\
& \omega_L \frac{\sin(\omega_L t)}{\omega_L^2 - \Omega^2} \sin(\Omega t) - \Omega \frac{\cos(\omega_L t)}{\omega_L^2 - \Omega^2} \cos(\Omega t) \Big) + O(t^{-1}) \\
tF_{\text{Ci}}(+, -, +, -) &= 2 \Big(\omega_L \frac{\sin(\omega_L t)}{\omega_L^2 - \Omega^2} \cos(\Omega t) - \Omega \frac{\sin(\omega_L t)}{\omega_L^2 - \Omega^2} \sin(\Omega t) - \\
& \omega_U \frac{\sin(\omega_U t)}{\omega_U^2 - \Omega^2} \cos(\Omega t) + \Omega \frac{\cos(\omega_U t)}{\omega_U^2 - \Omega^2} \sin(\Omega t) \Big) + O(t^{-1}) \\
tF_{\text{Ci}}(+, -, -, +) &= 2 \Big(\Omega \frac{\sin(\omega_L t)}{\omega_L^2 - \Omega^2} \cos(\Omega t) - \omega_L \frac{\cos(\omega_L t)}{\omega_L^2 - \Omega^2} \sin(\Omega t) + \\
& \omega_U \frac{\cos(\omega_U t)}{\omega_U^2 - \Omega^2} \sin(\Omega t) - \Omega \frac{\sin(\omega_U t)}{\omega_U^2 - \Omega^2} \cos(\Omega t) \Big) + O(t^{-1}).
\end{aligned} \tag{B.4}$$

Appendix C. Low temperature

Here we present details of computing $\langle\langle f_T^\Gamma(t; \omega) \rangle\rangle$ and $\langle\langle f_T^B(t; \omega) \rangle\rangle$ in the low temperature regime. As was already mentioned in the main text, in this case $\langle\langle f_T^\Gamma(t; \omega) \rangle\rangle \approx \langle\langle f_T^B(t; \omega) \rangle\rangle \approx \langle\langle f_0(t; \omega) \rangle\rangle$,

$$\begin{aligned}
\langle\langle f_0(t; \omega) \rangle\rangle &= \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} \int_{\omega_L}^{\omega_U} \frac{\omega_k}{(\omega_k^2 - \Omega^2)^2} \Big((1 - \cos^2 \Omega t) + \frac{\Omega^2}{\omega^2} (1 + \cos^2 \Omega t) \\
& - 2 \cos \Omega t \cos \omega t - \frac{2\Omega}{\omega_k} \sin \Omega t \sin \omega t \Big) = \\
& \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} (I_1 + I_2 - 2I_3 - 2I_4),
\end{aligned} \tag{C.1}$$

where the mean consists of four integrals. Results of each integration are given by:

$$\begin{aligned}
I_1 &= \int_{\omega_L}^{\omega_U} \frac{\omega}{(\omega^2 - \Omega^2)} (1 + \cos^2(\Omega t)) = \\
& \frac{1}{2} \left(\frac{1}{\omega_L^2 - \Omega^2} - \frac{1}{\omega_U^2 - \Omega^2} \right) (1 + \cos^2(\Omega t))
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
I_2 &= \int_{\omega_L}^{\omega_U} \frac{\Omega^2}{\omega(\omega^2 - \Omega^2)} (1 - \cos^2(\Omega t)) = \\
& \left[\frac{1}{4\Omega^2} \left(4 \log \frac{\omega_U}{\omega_L} - 2 \log \frac{\omega_U^2 - \Omega^2}{\omega_L^2 - \Omega^2} \right) + \frac{1}{2(\omega_L^2 - \Omega^2)} - \frac{1}{2(\omega_U^2 - \Omega^2)} \right] (1 - \cos^2(\Omega t))
\end{aligned} \tag{C.3}$$

$$\begin{aligned}
I_3 &= \int_{\omega_L}^{\omega_U} d\omega \frac{\omega}{(\omega^2 - \Omega^2)^2} \cos \omega t \cos \Omega t = \\
& \frac{1}{4\Omega} \cos \Omega t \left[\frac{2\Omega \cos \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\Omega \cos \omega_U t}{\omega_U^2 - \Omega^2} + \right.
\end{aligned} \tag{C.4}$$

$$\begin{aligned}
 & t \cos \Omega t F_{\text{Si}}(+, -, -, +) + t \sin \Omega t F_{\text{Ci}}(+, -, +, -) \Big] \\
 I_4 = & \int_{\omega_L}^{\omega_U} d\omega \frac{\Omega}{(\omega^2 - \Omega^2)^2} \sin \omega t \sin \Omega t = \\
 & \frac{1}{4\Omega} \sin \Omega t \left\{ \frac{2\omega_L \sin \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\omega_U \sin \omega_U t}{\omega_U^2 - \Omega^2} + \right. \\
 & t [\cos(\Omega t) F_{\text{Ci}}(-, +, -, +) + \sin(\Omega t) F_{\text{Si}}(+, -, -, +)] - \\
 & \left. \Omega^{-1} [F_{\text{Si}}(-, +, +, -) - F_{\text{Ci}}(-, +, -, +)] \right\}.
 \end{aligned} \tag{C.5}$$

To investigate the short-time behavior of $\langle\langle f_0(t; \omega) \rangle\rangle$ we expand the above expressions up to the second order in time. This is a good approximation for $t \ll \omega_U^{-1}$. As a result we obtain

$$\begin{aligned}
 I_1 = & \left[\frac{1}{4\Omega^2} \left(4 \log \frac{\omega_U}{\omega_L} - 2 \log \frac{\omega_U^2 - \Omega^2}{\omega_L^2 - \Omega^2} \right) + \frac{1}{2(\omega_L^2 - \Omega^2)} - \frac{1}{2(\omega_U^2 - \Omega^2)} \right] \Omega^2 t^2 + O(t^4)
 \end{aligned} \tag{C.6}$$

$$I_2 = \left(\frac{1}{\omega_L^2 - \Omega^2} - \frac{1}{\omega_U^2 - \Omega^2} \right) (2 - \Omega^2 t^2) + O(t^4) \tag{C.7}$$

$$\begin{aligned}
 I_3 = & \frac{1}{4\Omega} \left[\frac{2\Omega}{\omega_L^2 - \Omega^2} \left(1 - \frac{\omega_L^2 t^2}{2} - \frac{\Omega^2 t^2}{2} \right) - \right. \\
 & \left. \frac{2\Omega}{\omega_U^2 - \Omega^2} \left(1 - \frac{\omega_U^2 t^2}{2} - \frac{\Omega^2 t^2}{2} \right) + \Omega t^2 \log \frac{\omega_L^2 - \Omega^2}{\omega_U^2 - \Omega^2} \right] + O(t^4)
 \end{aligned} \tag{C.8}$$

$$I_4 = \frac{1}{4\Omega} \left(\frac{2\omega_L \omega_L t}{\omega_L^2 - \Omega^2} \Omega \omega_L t^2 - \frac{2\omega_U \omega_U t}{\omega_U^2 - \Omega^2} \Omega \omega_U t^2 \right) + O(t^4). \tag{C.9}$$

As a result, the expression for the mean valid in the short-time regime is

$$\langle\langle f_0(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} \log \left(\frac{\omega_U}{\omega_L} \right) t^2 + O(t^4), \tag{C.10}$$

what leads to (35) of text main text.

On the other hand, using asymptotic formulas from Appendix B.3 one can show that for $t \gg (\omega_L - \Omega)^{-1}$ $I_3 \approx 0$ and $I_4 \approx 0$ so the only relevant terms are I_1 and I_2 , what results in the following expression for long-time behavior of the mean

$$\langle\langle f_0(t; \omega) \rangle\rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\Delta\omega} (A_0 \cos^2(\Omega t) + B_0) + O(t^{-1}), \tag{C.11}$$

where

$$\begin{aligned}
 A_0 & \equiv -\frac{1}{2\Omega^2} \left(2 \log \frac{\omega_U}{\omega_L} - \log \frac{\omega_U^2 - \Omega^2}{\omega_L^2 - \Omega^2} \right), \\
 B_0 & \equiv \frac{1}{\omega_L^2 - \Omega^2} - \frac{1}{\omega_U^2 - \Omega^2} - A_0
 \end{aligned}$$

Minimization of (C.11) is straightforward, yields minimal value $\frac{2M\bar{\gamma}_0 B_0}{\hbar\pi\Delta\omega}$, what was used to derive formula (37) from text main text.

Appendix D. High temperature

In the case of high temperature one can approximate functions appearing in decoherence factor and generalized overlap as

$$f_T^\Gamma(t; \omega_k) \approx \frac{1}{\tau_T \omega_k} f_0(t; \omega_k), \quad f_T^B(t; \omega_k) \approx \tau_T \omega_k f_0(t; \omega_k).$$

Appendix D.1. Decoherence

We begin our considerations with the decoherence factor. The mean is given by an expression

$$\langle \langle f_T^\Gamma(t; \omega_k) \rangle \rangle = \quad (D.1)$$

$$\frac{2M\bar{\gamma}_0}{\hbar\pi\tau_T\omega_L\omega_U} \int_{\omega_L}^{\omega_U} \frac{1}{(\omega_k^2 - \Omega^2)^2} \left((1 - \cos^2 \Omega t) + \frac{\Omega^2}{\omega_k^2} (1 + \cos^2 \Omega t) \right. \\ \left. - 2 \cos \Omega t \cos \omega_k t - \frac{2\Omega}{\omega_k} \sin \Omega t \sin \omega_k t \right) = \quad (D.2)$$

$$\frac{2M\bar{\gamma}_0}{\hbar\pi\tau_T\omega_L\omega_U} (I_1 + I_2 - 2I_3 - 2I_4). \quad (D.3)$$

Computing integrals we obtain:

$$I_1 = \int_{\omega_L}^{\omega_U} d\omega \frac{\Omega^2}{\omega^2(\omega^2 - \Omega^2)^2} (1 - \cos^2(\Omega t)) = \quad (D.4)$$

$$\frac{(1 - \cos^2(\Omega t))}{\Omega^2} \left(\frac{\omega_U - \omega_L}{\omega_U \omega_L} - \frac{\omega_U}{2(\omega_U^2 - \Omega^2)} + \frac{\omega_L}{2(\omega_L^2 - \Omega^2)} + \frac{3}{4\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right)$$

$$I_2 = \int_{\omega_L}^{\omega_U} d\omega \frac{1}{(\omega^2 - \Omega^2)^2} (1 + \cos^2(\Omega t)) = \quad (D.5)$$

$$\frac{1}{4\Omega^2} (1 + \cos^2(\Omega t)) \left(\frac{2\omega_L}{\omega_L^2 - \Omega^2} - \frac{2\omega_U}{\omega_U^2 - \Omega^2} + \frac{1}{\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right)$$

$$I_3 = \int_{\omega_L}^{\omega_U} d\omega \frac{1}{(\omega^2 - \Omega^2)^2} \cos \omega t \cos \Omega t = \quad (D.6)$$

$$\frac{1}{4\Omega^2} \cos \Omega t \left[\frac{2\omega_L \cos \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\omega_U \cos \omega_U t}{\omega_U^2 - \Omega^2} + \right. \\ \left. t \cos(\Omega t) F_{\text{Si}}(+, -, +, -) + t \sin(\Omega t) F_{\text{Ci}}(+, -, -, +) + \right. \\ \left. \frac{1}{\Omega} (\cos(\Omega t) F_{\text{Ci}}(+, -, -, +) + \sin(\Omega t) F_{\text{Si}}(-, +, -, +)) \right]$$

$$I_4 = \int_{\omega_L}^{\omega_U} d\omega \frac{\Omega}{\omega(\omega^2 - \Omega^2)^2} \sin \omega t \sin \Omega t = \quad (D.7)$$

$$\frac{1}{2\Omega^3} \sin \Omega t \left[2 (\text{Si}(\omega_U t) - \text{Si}(\omega_L t)) - \right. \\ \left. \cos(\Omega t) F_{\text{Si}}(-, +, -, +) - \sin(\Omega t) F_{\text{Ci}}(-, +, +, -) \right. \\ \left. + \frac{\Omega}{2} \left(\frac{2\Omega \sin \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\Omega \sin \omega_U t}{\omega_U^2 - \Omega^2} + \right. \right.$$

$$t \cos(\Omega t) F_{\text{Ci}}(-, +, +, -) + t \sin(\Omega t) F_{\text{Si}}(+, -, +, -) \Big]$$

Now we turn our attention to short-time behavior of the mean. We expand the above expressions up to the second order in time. This is a good approximation for $t \ll \omega_U^{-1}$.

As a result we obtain

$$I_1 = \left(\frac{\omega_U - \omega_L}{\omega_U \omega_L} - \frac{\omega_U}{2(\omega_U^2 - \Omega^2)} + \frac{\omega_L}{2(\omega_L^2 - \Omega^2)} + \frac{3}{4\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right) t^2 + O(t^4) \quad (\text{D.8})$$

$$I_2 = \frac{1}{4\Omega^2} \left(\frac{2\omega_L}{\omega_L^2 - \Omega^2} - \frac{2\omega_U}{\omega_U^2 - \Omega^2} + \frac{1}{\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right) (2 - \Omega^2 t^2) + O(t^4) \quad (\text{D.9})$$

$$I_3 = \frac{\Omega^2}{4} \left(\frac{2\omega_L}{\omega_L^2 - \Omega^2} - \frac{2\omega_U}{\omega_U^2 - \Omega^2} + \frac{1}{\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right) \quad (\text{D.10})$$

$$+ \frac{1}{2(\omega_L^2 - \Omega^2)(\omega_U^2 - \Omega^2)} (\omega_L \Omega^2 + \omega_U \omega_L - \omega_L \omega_U^2 - \omega_U \Omega^2) t^2 + O(t^4)$$

$$I_4 = \left(\frac{1}{2\omega_L^2 - \Omega^2} - \frac{1}{2\omega_U^2 - \Omega^2} + \frac{1}{4\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right) t^2 + O(t^4) \quad (\text{D.11})$$

Finally, we can add the terms to obtain

$$\langle \langle f_T^\Gamma(t_S; \omega) \rangle \rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\tau_T\omega_L\omega_U} t^2 + O(t^4), \quad (\text{D.12})$$

what leads to the eq. (45) of the main text.

In the case of long-time behavior, one reaches a similar qualitative conclusion as for low-temperature case. Namely, for $t \gg (\omega_L - \Omega)^{-1}$ $I_3 \approx 0$ and $I_4 \approx 0$ so the only relevant terms are I_1 and I_2 , what results in the following expression for long-time behavior of the mean

$$\langle \langle f_T^\Gamma(t_{AS}; \omega) \rangle \rangle = \frac{2M\bar{\gamma}_0}{\hbar\pi\tau_T\Delta\omega} (A_\Gamma \cos^2(\Omega t) + B_\Gamma) + O(t^{-1}) \quad (\text{D.13})$$

where this time

$$A_\Gamma \equiv -\frac{1}{4\Omega^2} \left(\frac{\omega_U - \omega_L}{\omega_U \omega_L} + \frac{1}{2\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_U - \Omega)(\omega_L + \Omega)} \right),$$

$$B_\Gamma \equiv \frac{1}{4\Omega^2} \left(\frac{\omega_L}{(\omega_L^2 - \Omega^2)} - \frac{\omega_U}{(\omega_U^2 - \Omega^2)} \right) - A_\Gamma,$$

To obtain eq. (57) of the main text one performs straightforward minimization of (D.13).

Appendix D.2. Generalized overlap

In the case of generalized overlap the mean is given by

$$\langle \langle f_T^B(t; \omega_k) \rangle \rangle = \frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi\Delta\omega} \int_{\omega_L}^{\omega_U} \frac{\omega^2}{(\omega_k^2 - \Omega^2)^2} \left((1 + \cos^2 \Omega t) + \frac{\Omega^2}{\omega_k^2} (1 - \cos^2 \Omega t) \right) \quad (\text{D.14})$$

$$- 2 \cos \Omega t \cos \omega_k t - \frac{2\Omega}{\omega_k} \sin \Omega t \sin \omega_k t) =$$

$$\frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi\Delta\omega} (I_1 + I_2 - 2I_3 - 2I_4)$$

The results of integration are:

$$I_1 = \int_{\omega_L}^{\omega_U} d\omega \frac{\omega^2}{(\omega^2 - \Omega^2)^2} (1 + \cos^2 \Omega t) = \quad (D.15)$$

$$\frac{\omega_L}{2(\omega_L^2 - \Omega^2)} - \frac{\omega_U}{2(\omega_U^2 - \Omega^2)} + \frac{1}{4\Omega} \log \frac{(\omega_U - \Omega)(\omega_L + \Omega)}{(\omega_L - \Omega)(\omega_U + \Omega)}$$

$$I_2 = \int_{\omega_L}^{\omega_U} d\omega \frac{\Omega^2}{(\omega^2 - \Omega^2)^2} (1 - \cos^2 \Omega t) = \quad (D.16)$$

$$\frac{\omega_L}{2(\omega_L^2 - \Omega^2)} - \frac{\omega_U}{2(\omega_U^2 - \Omega^2)} + \frac{1}{4\Omega} \log \frac{(\omega_U + \Omega)(\omega_L - \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)}$$

$$I_3 = \int_{\omega_L}^{\omega_U} d\omega \frac{\omega^2}{(\omega^2 - \Omega^2)^2} \cos \omega t \cos \Omega t = \quad (D.17)$$

$$\frac{1}{4\Omega} \left(\frac{2\omega_L \cos \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\omega_U \cos \omega_U t}{\omega_U^2 - \Omega^2} + \right.$$

$$t(\cos(\Omega t) F_{\text{Si}}(+, -, +, -) + \sin(\Omega t) F_{\text{Ci}}(+, -, -, +)) +$$

$$\left. \frac{1}{\Omega} \cos(\Omega t) (\cos(\Omega t) F_{\text{Ci}}(-, +, +, -) + \sin(\Omega t) F_{\text{Si}}(+, -, +, -)) \right)$$

$$I_4 = \int_{\omega_L}^{\omega_U} d\omega \frac{\omega \Omega}{(\omega^2 - \Omega^2)^2} \sin \omega t \sin(\Omega t) = \quad (D.18)$$

$$\frac{1}{4} \sin(\Omega t) \left[\frac{2\Omega \sin \omega_L t}{\omega_L^2 - \Omega^2} - \frac{2\Omega \sin \omega_U t}{\omega_U^2 - \Omega^2} + \right.$$

$$\left. t(\cos(\Omega t) F_{\text{Ci}}(-, +, +, -) + \sin(\Omega t) F_{\text{Si}}(+, -, +, -)) \right]$$

With regard to short-time behavior of the mean, we expand the above expressions up to the second order in time. This is a good approximation for $t \ll \omega_U^{-1}$. As a result we obtain

$$I_1 = \quad (D.19)$$

$$\left(\frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} - \frac{1}{2\Omega} \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} \right) \frac{2 - \Omega^2 t^2}{2} + O(t^4)$$

$$I_2 = \quad (D.20)$$

$$\frac{\Omega^2}{2} \left(\frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} + \frac{1}{2\Omega} \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} \right) t^2 + O(t^4)$$

$$I_3 = \quad (D.21)$$

$$\frac{1}{2} \left(\frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} - \frac{1}{2\Omega} \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} \right) +$$

$$\frac{1}{4} \left(\frac{\omega_U}{\omega_U^2 - \Omega^2} (\omega_U^2 + \Omega^2) - \frac{\omega_L}{\omega_L^2 - \Omega^2} (\omega_L^2 + \Omega^2) + \right.$$

$$\left. 3(\omega_L - \omega_U) + 2\Omega \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} \right) t^2 + O(t^4)$$

$$I_4 = \frac{\Omega^2}{2} \left(\frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} - \frac{1}{2\Omega} \log \frac{(\omega_L - \Omega)(\omega_U + \Omega)}{(\omega_L + \Omega)(\omega_U - \Omega)} \right) t^2 + O(t^4) \quad (\text{D.22})$$

Finally, we can add the terms to obtain

$$\langle \langle f_T^B(t_S; \omega_k) \rangle \rangle = \frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi\Delta\omega} t^2 + O(t^4), \quad (\text{D.23})$$

what leads to the eq. (46) of the main text.

After taking into account that for $t \gg (\omega_L - \Omega)^{-1}$ $I_3 \approx 0$ and $I_4 \approx 0$ so the only relevant terms are I_1 and I_2 , one obtains the asymptotic formula for the mean

$$\langle \langle f_T^B(t; \omega) \rangle \rangle = \frac{2M\bar{\gamma}_0\tau_T}{\hbar\pi\Delta\omega} (A_B \cos^2(\Omega t) + B_B) + O(t^{-1}), \quad (\text{D.24})$$

with

$$A_B \equiv \frac{1}{2\Omega} \log \frac{(\omega_U - \Omega)(\omega_L + \Omega)}{(\omega_L - \Omega)(\omega_U + \Omega)} \quad (\text{D.25})$$

$$B_B \equiv \frac{\omega_L}{\omega_L^2 - \Omega^2} - \frac{\omega_U}{\omega_U^2 - \Omega^2} \quad (\text{D.26})$$

After straightforward minimization of (D.14) one arrives at eq. (58) of the main text.

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